Homework Assignment 2

Due by 5pm via Canvas, Thursday Feb 2nd

SDS 321 Intro to Probability and Statistics

1. A total of 50 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 20 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 60 percent of the Liberals, and 50 percent of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is:

   (a) (2 pts) What fraction of voters participated in the local election? $L := \{\text{Voter is liberal}\}$. $C := \{\text{Voter is conservative}\}$ and $I := \{\text{Voter is independent}\}$. $V = \{Voted\}$. $P(V) = P(V|C)P(C) + P(V|L)P(L) + P(V|I)P(I) = .5 \times .2 + .6 \times .3 + .35 \times .5 = .1 + .18 + .175 = .455$.

   (b) (1 pts) an Independent? $P(I|V) = \frac{P(V|I)P(I)}{P(V)} = \frac{.175}{.455} = .384$.

   (c) (1 pts) a Liberal? $P(L|V) = \frac{P(V|L)P(L)}{P(V)} = \frac{.18}{.455} = .396$.

   (d) (1 pts) a Conservative? $P(C|V) = \frac{P(V|C)P(C)}{P(V)} = \frac{.1}{.455} = .22$.

2. Alice is taking a pregnancy test. On an average, about 60% of women taking a pregnancy test are actually pregnant. The false positive rate is 1.5 percent and the false negative rate is 1 percent. $T = \{\text{Test is +ve}\}$, $Pr = \{\text{Alice is pregnant}\}$. $P(Pr) = .6$ and $P(T|Pr^c) = \text{false positive} = .015$ and $P(T^c|Pr) = .01$.

   (a) (2 pts) Alice takes the test and it comes out positive. Given this, whats the probability that Alice is pregnant?

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\begin{aligned}
P(Pr|T) &= \frac{P(T|Pr)P(Pr)}{P(T|Pr)P(Pr) + P(T|Pr^c)P(Pr^c)} \\
&= \frac{(1 - P(T^c|Pr))P(Pr)}{(1 - P(T^c|Pr))P(Pr) + P(T|Pr^c)P(Pr^c)} \\
&= \frac{.99 \times .6}{.99 \times .6 + .015 \times .4} = .99
\end{aligned}
$$

   (b) (4 pts) Alice takes the test again, and it comes out positive again. Given the results of the two tests what is the probability that she is pregnant? Use conditional independence.

$$
\begin{aligned}
P(Pr|T,T) &= \frac{P(T,T|Pr)P(Pr)}{P(T,T|Pr^c)P(Pr^c)+P(T,T|Pr)P(Pr)} \\
&= \frac{P(T|Pr)^2P(Pr)}{P(T|Pr)^2P(Pr)+P(T|Pr^c)^2P(Pr^c)} \\
&= \frac{.99^2\times .6}{.99^2\times .6+.015^2\times .4} = .9998
\end{aligned}
$$

(c) (4 pts) In the last question if Alice's second test comes out to be negative, then given results of the two tests (positive, negative) what is the probability that she is pregnant?

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\begin{aligned}
P(Pr|T,T^c) &= \frac{P(T,T^c|Pr)P(Pr)}{P(T,T^c|Pr^c)P(Pr^c)+P(T,T^c|Pr)P(Pr)} \\
&= \frac{P(T|Pr)P(T^c|Pr)P(Pr)}{P(T|Pr)P(T^c|Pr)P(Pr)+P(T|Pr^c)P(T^c|Pr^c)P(Pr^c)} \\
&= \frac{.99(1-.99)\times .6}{.99(1-.99)\times .6+.015(1-.015)\times .4} = .5
\end{aligned}
$$

3. Independent flips of a coin that lands on heads with probability 1/2 are made. What is the probability that the first four outcomes are

   (a) (1 pt) H, H, H, H? Using independence 1/16

   (b) (1 pt) T, H, H, H? Using independence 1/16

   (c) (3 pts) What is the probability that the pattern T, H, H, H occurs before the pattern H, H, H, H?

   Hint for part (c): How can the pattern H, H, H, H occur first? The key is to understand that if HHHH did not occur on the first 4 tosses, then for the first occurrence of HHHH down the sequence you must have had a T before the HHHH. Which means the only way HHHH can occur before THHH only such that HHHH occurs in the first 4 places. This is 1/16 and you are interested in $1 - 1/16 = 15/16$.