Homework Assignment 5

Due Oct 18

SDS 321 Intro to Probability and Statistics

1. (5 points) A man has 10 keys, one of which opens the front door. He picks one key at random (without replacement) and stops when he has found the right key. All keys are equally likely to be picked. There are 10 possible outcomes of this "experiment", i.e. {the first key is the right key}, {the second key is the right key},..., {the 10^{th} key is the right key}. Prove that each of these 10 outcomes has probability 1/10.

2. (1 + 1+ 2 + 1 + 3 points) Consider a Bernoulli random variable X such that $P(X = 1) = p$. Calculate the following:

   (a) $E[(1 - X)^{10}]$

   (b) $E[(X - p)^4]$.

   (c) $\text{var}((X - p)^2)$.

   (d) $E[3^X 4^{1-X}]$.

   (e) $\text{var}(3^X 4^{1-X})$.

3. (3+2) The PMF of a random variable is given by:

$$P(X = x) = \begin{cases} 2a & \text{x=0} \\ b & \text{x=1} \\ a & \text{x=2} \end{cases}$$

   We also have $E[X] = 4/5$.

   (a) What are a and b?

   (b) What is $\text{var}(X)$?

   3. (3pts) If a coin is tossed a sequence of times (infinitely many times), what is the probability that the first head will occur **after** the 5-th toss, given that it has not occurred in the first 2 tosses?

   In your solution let $A =$"first head after 5th toss"; and $B =$"no head in first 2 tosses"

4. Charles claims he can distinguish between beer and ale 75% time. Let p = Charles' probability of distinguishing the drinks. Charles' claim is $p = 0.75$. Ruth bets that he cannot and, in fact, just guesses. That is, Ruth's claim is $p = 0.5$. To settle this, a bet is made: Charles is to be given $n = 5$ small glasses, each having been filled with beer or ale, chosen by tossing a fair coin. He wins the bet if he gets 4 **or more** correct. In your answer let X ="number of glasses he gets correct", and use $p_1 = 0.75$ and $p_0 = 0.5$.

   (a) (3pts) Find the probability that Charles wins if his claim is right, that is, if $p = p_1$.

   (b) (3pts) Find the probability that Charles wins if his claim is wrong, that is, if $p = p_0$.

   (c) (3pts) Assume that you believe that Charles is right with probability 0.1, and Ruth is right with probability 0.9 (that is, before you see the outcome of the bet). Given that Charles gets $X = 4$ correct, what is the probability that his claim is right? In your answer let A ="Charles is right" (that is, $p = p_1$), and B ="Charles gets 4 correct". *Hint:* A^c = "Ruth is right".