

# Homework Assignment 6

Due **Sunday** Nov 12 by 6pm

SDS 321 Intro to Probability and Statistics

1. **(1+2+2 pts)** A continuous random variable  $X$  has PDF

$$f_X(x) = \begin{cases} c(2 + x^2) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is the value of  $c$ ?
- (b) What is the CDF,  $F_X(x)$ , of  $X$ ? Be sure to specify  $F_X(x)$  for all values of  $-\infty < X < \infty$ .
- (c) What is the expectation and variance of  $X$ ?
2. **(1+1+1+2 pts)** Based on US population statistics, we can model the height of a randomly selected man using a normal random variable with mean 69.1 inches and standard deviation 2.9 inches. We can model the height of a randomly selected woman using a normal random variable with mean 63.7 inches and standard deviation 2.7 inches.
- (a) Write the probability of a man being between 6' and 6'2" tall, in terms of the CDF of the standard normal distribution,  $\Phi(z) = \mathbf{P}(Z \leq z)$  for  $Z \sim N(0, 1)$ .
- (b) Using the look-up table for the standard normal table (see attached table) and your answer from [a], what is the probability of a man being between 6' and 6'2" tall?
- (c) What is the probability of a randomly selected individual being less than 5'6" tall? You may assume that  $\mathbf{P}(\text{man}) = \mathbf{P}(\text{woman}) = 0.5$ .
- (d) Given the fact that a randomly selected individual is shorter than 5'5", what is the probability that the individual is a man?
3. **(2+2)** You throw a dart that is equally likely to fall anywhere in the range  $[0, 1]$ . Let  $X$  be the distance of the dart from 0.
- (a) What is the distribution of  $1 - X$ ?
- (b) What is the probability that the ratio of the length of the shorter segment to that of the longer segment is less than  $1/4$ ?
4. **(1+2+1+2 pts)** **[Exponential distribution]** I buy a new laptop. According to the manufacturer, the lifetime,  $X$ , of the laptop is a continuous random variable with PDF  $f_X(x) = 0.5e^{-0.5x}$ , for all  $x \geq 0$ .  $f_X(x) = 0$  for all  $x < 0$ .
- (a) What is the probability that the laptop fails during the first year?
- (b) What is the expected lifetime of the laptop?
- (c) 3 years pass, and the laptop is still going strong. What is the conditional PDF of the total lifetime, given I know the lifetime is at least 3 years?
- (d) Based on this conditional PDF, how many more years do I expect it to last?

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990