Practice Quiz 1

SDS 321

1. $P(A) = 0.40$, and $P(B) = 0.35$. Find $P(A \cup B)$.

   (a) If A and B are independent.

   If A and B are independent then $P(A \cap B) = P(A) \cdot P(B)$, so $P(A \cap B) = 0.40 \cdot 0.35 = 0.14$. Then, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.35 - 0.14 = 0.61$

   (b) If A and B are mutually exclusive.

   If A and B are mutually exclusive then $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B) = 0.40 + 0.35 = 0.75$

2. I flip a coin twice, recording the results for each flip. If I consider the first flip distinct from the first, answer the following questions.

   (a) What is the sample space for this experiment?

   $$\{HH, HT, TH, TT\}$$

   (b) Which of the following are valid partitions of the sample space, why or why not?

      i. $\{\{H\}, \{T\}\}$ No, not descriptive enough

      ii. $\{\{HH, HT\}, \{TH, TT\}\}$ Yes

      iii. $\{\{HH\}, \{TH, TT\}\}$ No, missing HT.

3. I roll two fair dice, one is red and one is white. Consider the following events:

   - A: Red die outcome is odd,
   - B: Red die outcome is ≤ 3,
   - C: Sum of the dice rolls is 9.

   Answer the following questions:

   (a) How many possible outcomes are there in the sample space (Ω)?

   $$36$$

   (b) What are the outcomes for event C?

   $$\{(3,6),(6,3),(4,5),(5,4)\}$$

   (c) What are the outcomes for $A \cap B \cap C$?

   $$\{(3,6)\}$$

   (d) Are A, B, and C pairwise independent?

   $$P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$
   $$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$
   $$P(A \cap B) \neq P(A) \cdot P(B)$$

   No they are not pairwise independent.

   (e) Are A, B, and C mutually independent?

   Since they are not pairwise independent they are not mutually independent.

4. A researcher is interested in the effects of different fertilizers on flowering plants. They apply fertilizer A to 20% of plants and fertilizer B to 20%, leaving the remaining 60% unfertilized. The probability of a plant flowering in fertilizer A is 0.75, the probability of a plant flowering in fertilizer B is 0.85, and the probability of a plant flowering in unfertilized soil is 0.6, what is the overall probability of flowering?

   Let:

   - F: flowers
   - A: fertilizer A
   - B: fertiliser B
   - U: unfertilized

$$\begin{aligned} P(F) &= P(F \mid A)P(A) + P(F \mid B)P(B) + P(F \mid U)P(U) \\ &= 0.75 \cdot 0.20 + 0.85 \cdot 0.20 + 0.60 \cdot 0.60 \\ &= 0.15 + 0.17 + 0.36 = 0.68 \end{aligned}$$

5. A student has two statistics exams this week. They have an equal probability of passing each exam. The probability they pass at least one exam is 0.9. The probability they pass both exams is 0.7.

   (a) What is the marginal probability they pass Exam 1?

$$\begin{aligned} P(\text{Exam 1} \cup \text{Exam 2}) &= P(\text{Exam 1}) + P(\text{Exam 2}) - P(\text{Exam 1} \cap \text{Exam 2}) \\ 0.9 &= 2 \cdot P(\text{Exam 1}) - 0.7 \\ P(\text{Exam 1}) &= 0.8 \end{aligned}$$

   (b) Given they pass Exam 1 what is the probability they pass Exam 2?

$$\begin{aligned} P(\text{Exam 2} \mid \text{Exam 1}) &= \frac{P(\text{Exam 1} \cap \text{Exam 2})}{P(\text{Exam 1})} \\ &= \frac{0.7}{0.8} = 0.875 \end{aligned}$$

6. Let $P(A) = 0.47$, $P(C \mid A) = 0.52$, $P(B) = 0.32$, $P(B \cap C) = 0.21$, and $P(C) = 0.61$.

   (a) What is $P(A \mid C)$?

$$\begin{aligned} P(A \mid C) &= \frac{P(A \cap C)}{P(C)} \\ &= \frac{P(C \mid A)P(A)}{P(C)} \\ &= \frac{0.52 \cdot 0.47}{0.61} = \frac{611}{1525} \approx 0.40 \end{aligned}$$

   (b) What is $P(B \mid C)$?

$$\begin{aligned} P(B \mid C) &= \frac{P(B \cap C)}{P(C)} \\ &= \frac{0.21}{0.61} = \frac{21}{61} \approx 0.34 \end{aligned}$$

   (c) What value of $P(A \cap B)$ would make A and B independent?

$$P(A \cap B) = P(A)P(B) = 0.47 \cdot 0.32 = 0.1504$$

   (d) What value of $P(A \cap B \cap C)$ would make A and B conditionally independent given C?

$$\begin{aligned} P(A \cap B \mid C) &= P(A \mid C)P(B \mid C) \\ &= \frac{611}{1525} \cdot \frac{21}{61} = \frac{12831}{93025} \end{aligned}$$

   Then:

$$P(A \cap B \cap C) = P(A \cap B \mid C)P(C) = \frac{12831}{93025} \cdot \frac{61}{100} \approx 0.08$$

7. A woman takes a pregnancy test. The test has a false positive rate of 0.025, and a false negative rate of 0.01. If the probability that she is pregnant is 0.6, what is the probability she is pregnant given the pregnancy test is positive?

   Let:

- P: pregnant
- P^c: not pregnant
- $+$: test positive
- $-$: test negative

$$\begin{aligned} P(P \mid +) &= \frac{P(+ \mid P)P(P)}{P(+ \mid P)P(P) + P(+ \mid P^c)P(P^c)} \\ &= \frac{0.99 \cdot 0.6}{0.99 \cdot 0.6 + 0.025 \cdot 0.4} \\ &\approx 0.98 \end{aligned}$$

8. How many way can I permute the letters in the following words?

   (a) chanterelle

   $$\frac{11!}{3!2!} = 3326400$$

   (b) shiitake

   $$\frac{8!}{2!} = 20160$$

   (c) oyster

   $$6! = 720$$

9. A parent is hosting a Halloween party and bought 20 total pieces of candy. If 12 total children show up, how many ways can they distribute the candy to the children in the following scenarios?

   (a) All of the pieces of candy are identical, but each child has to receive at least one piece of candy.

   $$\binom{20-1}{12-1} = \binom{19}{11} = \frac{19!}{11!8!} = 75582$$

   (b) All of the pieces of candy are different, but each child receives a single piece of candy.

   $$P_{12}^{20} = \frac{20!}{8!}$$