

SDS321 Practice Problems for Exam 1

- A bag contains 8 pairs of shoes; each pair is a different style. You pick two random shoes from the bag.
 - What is the probability that the two shoes you picked out are a pair; i.e. left and right of the same style?
 - What is the probability you picked one left shoe and one right shoe?
- How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 11$, where x_1, x_2, x_3 are non-negative integers.
- Suppose a department has 10 men and 15 women. How many ways can we form a committee with six members, if it must have more women than men?
- How many ways are there to choose a half dozen donuts from 10 varieties
 - If there are no two donuts of the same variety.
 - If there are at least two varieties.
 - If there must be at least one but no more than 4 glazed.
- Given a set $A = \{a,b,c,d,e\}$,
 - Suppose you create sequences by drawing from A with replacement. How many different sequences of type A of length $n > 0$ exist that contain at most one a ?
 - How many subsets of A are there?
 - How many non-empty subsets of A are there?
 - How many subsets of size 3 can you create from A ?
 - How many subsets of A are there that are entirely vowels or entirely consonants? (Assume empty can be considered as entirely vowels or as entirely consonants).
 - How many subsets of A are there that have at least one vowel and one consonant?
 - How many subsets of A of size 3 contain exactly one vowel.
 - How many ways can you arrange the letters in A ?
 - How many ways can the letters of A be arranged so that all of the vowels are together?
 - How many ways can you arrange the letters of A so that it is not the case that all of the vowels are together?
 - How many ways can you arrange the letters in A so that vowels and consonants alternate and the arrangement begins with a consonant.
 - For $n > 0$, assume all strings of length n from the set A (allowing repetition) are equally likely.
 - What is the probability that such a string has no a ?
 - What is the probability that such a string has no b given that it has no a ?
- How many distinct permutations are there of the letters in “perfect”?

7. Billy takes two tests in his probability class. The probability that Billy would pass at least one test is 0.9. The probability that he passes both tests is 0.7. The tests are of equal difficulty (that is, the probability that Billy passes test 1 is the same as the probability that he passes test 2.) What is the conditional probability of Billy passing test 2 given the event that he passes test 1?
8. Three persons roll a fair 4-sided die once. Let B_{ij} be the event that person i and person j roll the same face. Show that the events B_{12} , B_{13} , and B_{23} are pairwise independent but are not independent.
9. Let $P(A)=.5$, $P(B)=.6$, and $P(A \cap B^c)=.2$. Are A and B independent? What is $P(A \cap B | A \cup B)$?
10. If $P(A)=0.4$, $P(B)=0.3$ and $P((A \cup B)^c)=0.42$. Are A and B independent?
11. Two fair dice are rolled. Define events: A =sum of two dice equals 3, B =sum of two dice equals 7, C =at least one of the dice shows a 1.
What is $P(A|C)$? What is $P(B|C)$? Are A and C independent?
12. Four graduating seniors Alice, Bob, Charles and David are scheduled for job interviews at Random Sampling, Inc at the same time. They've been assigned by the personnel manager to four separate rooms. But they are not aware of this, and randomly assign themselves to the four rooms. What's the probability that none of them ends up in the correct room?
13. Three molecules of type A , three of type B , and three of type C are to be linked together to form a chain molecule. One example of such chain molecule is $ABCABCABC$.
 - (a) How many such chain molecules are there? (molecules of same type are indistinguishable.)
 - (b) If the link between consecutive molecule A s is unstable, how many stable chain molecules can be formed?
14. On Alex's commuting route there are three intersections with traffic signals. The probability that she must stop at the first signal is 0.4. If she stops at one signal, then the next signal is always green when she gets there. If she doesn't stop at one signal, the probability of her stopping at the next signal is 0.5.
 - (a) What is the probability she arrives school without stopping at signals.
 - (b) What is the probability that she stops at exactly one signal given the second signal is green?

15. Alex is new to basketball. He is learning to shoot the ball through the hoop from the free throw line, which he is currently able to do 20% of the time. He goes to the gym with the intention of practicing just his free throws.
- a) Suppose he is unable to get the ball through the hoop in his first 12 attempts. What is the probability that he will be able to make it for the first time, on his 15th attempt?
 - b) Suppose he is unable to get the ball through the hoop in his first 12 attempts. What is the probability that he will need at least 15 attempts to make his first successful shot?
16. Suppose 20% of people in a city are estimated to have a disease. You test 20 individuals chosen at random to see if they have the disease. What is the probability that at least two of the people test positive for the disease?
17. The number of typos in textbooks printed by publishing company XYZ follows a Poisson distribution. It is known that the mean number of typos in 100 pages of their textbooks is 3.2. What is the probability of exactly 10 typos in a 200-page textbook?