

SDS 321: Practice questions

1. How many unique combinations can you get by rearranging the letters MISSISSIPPI such that the two P's cannot appear together?

11 letters so 11! permutations. 4 Is, 4Ss, 2 Ps, so $11!/(4!4!2!)$ unique combinations. But now subtract the case with PP always appear together, so treat it as a unit, and count $10!/(4!4!)$. So the answer is $11!/(4!4!2!) - 10!/(4!4!) = 28350$

2. Combinatorics question:

- (a) How many different solutions are there to the equation $x_1 + x_2 + x_3 = 10$, where x_1, x_2 and x_3 are positive integers? (count " $x_1 = 1, x_2 = 2, x_3 = 7$ " and " $x_1 = 2, x_2 = 1, x_3 = 7$ " as two separate solutions). Stars and bars... We have 9 places to put the first bar, and 8 places to put the second. But, there are 2 possible rearrangements of the bars. So, $9 \times 8/2 = 36$. Or, equivalently, there are $\binom{9}{2=36}$ ways of placing 2 bars.
- (b) How many different solutions are there to the equation $x_1 + x_2 + x_3 = 10$, where $x_1 < x_2$? We know there are 36 solutions in total. Of these, let's remove the solutions where $x_1 = x_2$. We have 4 such solutions ($x_1 = x_2 = 1, x_1 = x_2 = 2, x_1 = x_2 = 3, x_1 = x_2 = 4$), leaving 32 solutions with different x_1 and x_2 . Of these, half have $x_1 < x_2$, so 16 solutions.
- (c) How many different solutions are there to the equation $x_1 + x_2 + x_3 = 10$, where $x_1 < x_2 < x_3$? We know there are 36 solutions in total. Let's first remove the solutions with repeats. We have no solutions with all three numbers the same, and 12 solutions with 2 numbers the same (3 ways each of having 2 ones, 2 twos, 2 threes, 2 fours). So, 24 solutions with no repeats. Of these, there are 3! permutations of each sequence, so there are $24/3! = 4$ solutions. Double checking, we have $1+2+7, 1+3+6, 1+4+5, 2+3+5$... that's it!

3. Defective parts from mixed suppliers.

A factory buys bolts from supplier A half the time, supplier B 30 percent of the time and supplier C the rest of the time. These have defect rates 2, 3 and 5%.

- (a) What is the probability that a bolt picked at random will be defective?

By the Total Probability Rule,

$$P(D) = P(D | A)P(A) + P(D | B)P(B) + P(D | C)P(C).$$

Substituting,

$$P(D) = 0.02(0.50) + 0.03(0.30) + 0.05(0.20) = 0.01 + 0.009 + 0.01 = 0.029.$$

- (b) Say you picked a bolt at random and it came out to be defective. What is the probability that it came from supplier A ?

i. **Find $P(A | D)$.**

We already have

$$P(A) = 0.50, \quad P(D | A) = 0.02, \quad P(D) = 0.029.$$

By Bayes' Law,

$$P(A | D) = \frac{P(D | A) P(A)}{P(D)}.$$

Substitute:

$$P(A | D) = \frac{0.02(0.50)}{0.029} = \frac{0.01}{0.029} = \frac{10}{29} \approx 0.3448.$$

4. I have three envelopes, each containing two objects. In one, there is a silver square and a gold disk. In another, there are a gold square and a gold disk. In the third, there are a silver square and a silver disk.

(a) I pick an envelope (at random), and take out an object (at random). It is gold. What is the probability that the second object is silver?

$$P(S2|G1) = P(G1, S2)/P(G1) = (1/6)/(1/2) = 1/3$$

(b) I put the objects back in their envelope and shuffle the envelopes. I again pick an envelope (at random), and take out an object (at random). It is a gold disk. What is the probability that the second object is silver?

$$P(SS2|GD1) = P(GD1, SS2)/P(GD1) = 1/6/1/3 = 1/2$$

5. My partner and I are one of 10 married couples at a dinner party. The 20 people are given random seats around a round table.

(a) What is the probability that I am sat next to my spouse? **there are two seats next to me, and 19 people who can sit in those seats. $P(\text{spouse on left}) = 1/19$, $P(\text{spouse on right} | \text{spouse not on left}) P(\text{spouse not on left}) = 1/18 \times 18/19 = 1/19$, so $P(\text{next to spouse}) = 2/19$.**

(b) What is the probability that I am seated between a person and their spouse? **there are 18 ways to pick your neighbor (they cannot be you or your husband) and then there are $17!$ ways to arrange the rest. So in total $18 \times 17!/19! = 1/19$.**