SDS 321: Practice questions

1. How many unique combinations can you get by rearranging the letters MISSISSIPPI?

2. On the first day of a non-leap-year, I put \$1 in a box. On the second day, I put \$2 in the box. On the third day, I put \$3 in. And so on. At the end of the year (365 days), how much money is in the box?

3. Let X be a normal random variable with mean 3 and variance 1, and let Y be a normal random variable with mean 4 and variance 2.

   (a) What is the distribution of $Z = X + Y$?

   (b) What is the probability that Z is between 6 and 8?

4. I am waiting for a bus, that I know will arrive at some time between 1pm and 2pm, with all times being equally likely. It gets to 1:30, and the bus has still not arrived. What is the probability that it arrives before 1:40?

5. Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 0.125x + 0.125 & -1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

   What is the PDF of $Z = |X|$?

6. Alice and Bob are playing rock-paper-scissors. If both Alice and Bob play the same hand, they play again. What is the expected number of turns before someone wins?

7. On a given day, a $Poisson(100)$ number of insects fly through my yard. Using an appropriate approximation, what is the the probability that, over the month of May (31 days), the average number of insects is between 98 and 102? You may use the fact that a $Poisson(\lambda)$ random variable has mean and variance λ.

8. Combinatorics question:

   (a) How many different solutions are there to the equation $x_1 + x_2 + x_3 = 10$, where x_1, x_2 and x_3 are positive integers? (count "$x_1 = 1, x_2 = 2, x_3 = 7$" and "$x_1 = 2, x_2 = 1, x_3 = 7$" as two separate solutions).

   (b) How many different solutions are there to the equation $x_1 + x_2 + x_3 = 10$, where $x_1 < x_2$?

   (c) How many different solutions are there to the equation $x_1 + x_2 + x_3 = 10$, where $x_1 < x_2 < x_3$?

9. Let X be a continuous random variable, with PDF:

$$f_X(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \leq x < 1 \\ ce^{-x} & x \geq 1 \end{cases}$$

   (a) What is c?

   (b) What is the conditional expectation of X, given $X < 1$?

   (c) What is the conditional expectation of X, given $X \geq 1$?

   (d) What is the expectation of X?

10. Let X and Y be random variables with joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} \frac{ay}{x^2} & x \geq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is a?

   (b) What is the conditional PDF $f_{Y|X}(y|x)$ of Y given $X = x$?

   (c) What is the conditional expectation of Y given X?

   (d) What is the expected value of Y?

11. I have three envelopes, each containing two objects. In one, there is a silver square and a gold disk. In another, there are a gold square and a gold disk. In the third, there are a silver square and a silver disk.

   (a) I pick an envelope (at random), and take out an object (at random). It is gold. What is the probability that the second object is silver?

   (b) I put the objects back in their envelope and shuffle the envelopes. I again pick an envelope (at random), and take out an object (at random). It is a gold disk. What is the probability that the second object is silver?

12. Let X and Y be uniform random variables between 0 and 1. What is the probability that:

   (a) $X < Y$

   (b) $X < 2Y$

   (c) $X + Y < 0.5$

   (d) $\max\{X, Y\} \leq 0.7$

13. If there are no distractions, it takes me 30 minutes to walk to the store. However, if I pass someone with a dog, I stop and pet the dog and chat to the owner. The number Y of dogs I pass is a Poisson random variable with mean 2. Each time I stop, the number of minutes I spend petting the dog and chatting is an exponential random variable with PDF:

$$f_X(x) = 0.5e^{-0.5x}$$

   (a) If I see a single dog, what is the expectation and variance of the time spent petting the dog and chatting to its owner?

(b) What is the conditional expectation of the total time spent petting dogs and chatting to their owners, as a function of Y?

(c) What is the conditional variance of the total time spent petting dogs and chatting to their owners, as a function of Y?

(d) What is the expectation and variance of the total time it takes me to get to the store? You may use the fact that the variance of a Poisson random variable is the same as its mean.

14. My partner and I are one of 10 married couples at a dinner party. The 20 people are given random seats around a round table.

(a) What is the probability that I am sat next to my spouse?

(b) What is expected number of couples that are sat next to each other?

Standard normal table

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990